WEEKLY TEST MEDICAL PLUS - 01 TEST - 21 R & B SOLUTION Date 13-10-2019

[PHYSICS]

1. $PV = RT$ (ideal gas equation) (i)

$V^2P = C$ (additional law) (ii)

Dividing $V = \frac{C}{RT} \Rightarrow V \propto \frac{1}{T}$

or $T \propto \frac{1}{V}$ (combined law)

When volume (V) is doubled, temperature is halved

i.e., $T' = \frac{T}{2}$

2. Total volume is halved. Pressure is doubled.

3. $T_1 = 300$ K, $P \propto T$

So, $T_2 = 600$ K or 327°C

4. $P = \frac{1}{3}dC^2$

or $dc^2 =$ constant or $dT =$ constant

5. For a constant value of density, pressure is more at T_1.

$\therefore$ $T_1 > T_2$ $[\because P \propto T]$

6. $VP^3 = \text{constant} = k \Rightarrow P = \frac{k}{V^{1/3}}$

Also $PV = \mu RT \Rightarrow \frac{k}{V^{1/3}} \cdot V = \mu RT \Rightarrow V^{2/3} = \frac{\mu RT}{k}$

Hence $\left(\frac{V_1}{V_2}\right)^{2/3} = \frac{T_1}{T_2} \Rightarrow \left(\frac{V}{27V}\right)^{2/3} = \frac{T}{T_2} \Rightarrow T_2 = 9T$

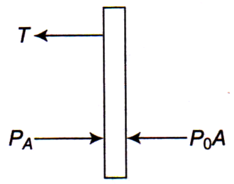
7. $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT_1} = \frac{P'v}{RT_1} + \frac{P'V}{RT_2} \Rightarrow \frac{2P}{RT_1} = \frac{P'}{R}\left[\frac{T_2 + T_1}{T_1T_2}\right]$$

$$P' = \frac{2PT_2}{(T_1 + T_2)} = \frac{2 \times 1 \times 600}{(300 + 600)} = \frac{4}{3}\text{atm}$$

8. $T + P_0A = PA$

$$T = (P - P_0)A = \frac{3}{8}P_0A \text{ (given)}$$



$$P = \frac{3}{8}P_0 + P_0 = \frac{11}{8}P_0$$

Now volume is constant by string

So, $P \propto T$

Initial temperature is T_0.

9. Initial volume of gas = V_1

Final volume of gas = V_2

Initial temperature of gas $T_1 = 27°C = 300$ K

Final temperature of gas $T_2 = 54°C = 327$ K

Now from the Charles's law at constant pressure

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} = \frac{300}{327} = \frac{100}{109}$$

10. $$\rho = \frac{PM}{RT}$$

ρ = constant, if (P/T) is constant

(P/T) is constant in option (d).

11. $$V = k\left(\frac{nRT}{VT}\right)^{0.33}$$

$V^{1.33}$ = constant

V = constant

12. $$\rho = \frac{PM}{RT} \text{ gives } \rho \propto (P/T)$$

$$\rho_A = \rho_0 \text{ and } \rho_B = \frac{3}{2}\rho_0$$

$$(P/T)_B = \frac{3}{2}(P/T)_A$$

$$(X/2T_0) = \left(\frac{3}{2}\right)(P_0/T_0)$$

$$X = 3P_0$$

13. $P = P_0[1 + (2V_0/V)^2 T^{-1}$

At $V = V_0, P = P_0/5$

$$T_i = \frac{PV}{nR} = \frac{(P_0/5)V_0}{nR} = \frac{P_0V_0}{5nR}$$

At $V = 2V_0, P = P_0/2$

$$T_f = \frac{PV}{nR} = \frac{(P_0/2)(2V_0)}{nR} = \frac{P_0V_0}{nR}$$

$$\Delta T = T_f - T_i = \frac{4P_0V_0}{5nR}$$

14. On combining the two vessels the total number of moles remains constant, i.e., $n = n_1 + n_2$

Using gas equation, we can write, $n_1 = \frac{P_1V}{RT_1}; n_2 = \frac{P_2V}{RT_2}$

and $n = \frac{P(2V)}{RT}$

where V is the volume of each vessel.

Thus $\frac{P(2V)}{RT} = \frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}$ or $\frac{P}{T} = \frac{1}{2}\left[\frac{P_1}{T_1} + \frac{P_2}{T_2}\right]$

15. $dQ = du + dW$ or $Q = (u_2 - u_1) + W$

$W = Q_{1b2} - (u_2 - u_1)$

or $Q_{1b2} - W = u_2 - u_1 = 36 - 30 = 6$ cal

or $Q_{1b2} - W = u_2 - u_1 = 36 - 30 = 6$ cal

16. $\Delta U = nC_V\Delta T = n(5/2)R\Delta T$

$\Delta Q = nC_P\Delta T = n(7/2)R\Delta T$

$$W = \Delta Q - \Delta U = \frac{n7}{2}R\Delta T - \frac{n5}{2}R\Delta T = nR\Delta T$$

$$\frac{W}{\Delta U} = \frac{2}{7}$$

17. Work done = Area between p–V graph and V-axis.

18. $Q_1 = mC_PdT$ at constant pressure.

Q_2 = Heat for internal energy = $m\,Cv\,dT$

$\therefore \frac{Q_2}{Q_1} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{1}{5/3} = 3/5$ for monoatomic gas.

19. Work done = Area under the P–V curve

$$W = (80\text{ kPa})(250 \times 10^{-6})kt\ 1/2 = 10\text{ J}$$

Since the arrow is anticlockwise,

$\therefore$ Work done = –10 J

20. $Q = nC_P\Delta T = 1 \times \frac{5}{2}R \times 10 = 25$ R

$$Q' = 1 \times \frac{7}{2}R \times 5 = \frac{35}{2}R$$

$$\frac{Q'}{Q} = \frac{35}{50} = \frac{7}{10} \Rightarrow Q' = \frac{7}{10}Q$$

21.

$$W = \text{Area } ABCDA = \frac{\pi R^2}{2}$$

$$= \frac{11 \times (20)^2}{2} = 200\,\pi \text{ joule}$$

22.

$$U = n\frac{F}{2}RT = 2 \times \frac{5}{2}RT + 4 \times \frac{3}{2}RT = 11RT$$

23.

Use $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

or $T_2 = \frac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{273}{(2)^{0.4}} = 207 \text{ K}$

Change in internal energy

$$\Delta U = \frac{R}{(\gamma - 1)}(T_1 - T_2) = \frac{8.31(273 - 207)}{1.4 - 1} = 1369.5 \text{ J}$$

24.

$$\Delta U = \Delta Q - p\Delta V = \Delta Q - pV$$

25.

Process AB is isochoric, $\therefore W_{AB} = P\Delta V = 0$

Process BC is isothermal $\therefore W_{BC} = RT_2 \cdot \ln\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric

$\therefore W_{CA} = -P\Delta V = -R\Delta T = -R(T_1 - T_2) = R(T_2 - T_1)$

(Negative sign is taken because of compression)

26. .

$W_{BCOB} = -$ Area of triangle $BCO = -\frac{P_0 V_0}{2}$

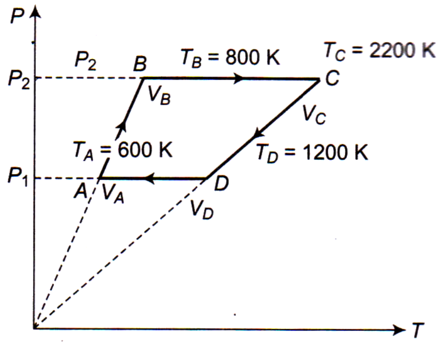
$W_{AODA} = +$ Area of triangle $AOD = +\frac{P_0 V_0}{2}$

27.

Processes A to B and C to D are parts of straight line graphs of the form $y = mx$

Also $P = \frac{\mu R}{V} T$ $(\mu = 6)$

$\Rightarrow$ $P \propto T$. So volume remains constant for the graphs AB and CD



So no work is done during processes for A to B and C to D i.e., $W_{AB} = W_{CD} = 0$ and $W_{BC} = P_2(V_C - V_B) = \mu R$ $(T_C - T_B)$

$$= 6R(2200 - 800) = 6R \times 1400\ J$$

Also $W_{DA} = P_1(V_A - V_D) = \mu R(T_A - T_B)$

$$= 6R(600 - 1200) = -6R \times 600 \text{ J}$$

Hence work done in complete cycle

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 0 + 6R \times 1400 + 0 - 6R \times 600$$

$$= 6R \times 900 = 6 \times 8.3 \times 800 \approx 40 \text{ kJ}$$

28.

$W_{AB} = -P_0V_0$, $W_{BC} = 0$ and $W_{CD} = 4P_0V_0$

$\Rightarrow \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

29.

$$W_{AB} = -\left(P_0V_0 + \frac{P_0V_0}{2}\right) = -\frac{3}{2}P_0V_0$$

$$W_{BC} = (2P_0)(2V_0) + \frac{P_0(2V_0)}{2} = 5P_0V_0$$

$$W_{ABC} = \frac{7}{2}P_0V_0$$

30.

The work done by the gas

$$W = \int_i^f PdV$$

$$W = \int_i^f \alpha V^2 dV = \frac{1}{3}\alpha(V_f^3 - V_i^3)$$

$$V_f = 2V_i = 2(1.00 \text{ m}^3) = 2.00 \text{ m}^3$$

$$W = \frac{1}{3}[(5.00 \text{ atm/m}^6)(1.013 \times 10^5 \text{ Pa/atm})]$$

$$\times [(2.00 \text{ m}^3)^3 - (1.00 \text{ m}^3)^3] = 1.18 \text{ MJ}$$

31. ΔU remains same for both path

For path *iaf*: $\Delta U = \Delta Q - \Delta W = 50 - 20 = 30$ J.

For path *fi*: $\Delta U = -30$ J and $\Delta W = -13$ J

$\Rightarrow \quad \Delta Q = -30 - 13 = -43$ J.

32. Change in internal energy from $A \to B$ is

$$\Delta U = \frac{f}{2}\mu R \Delta T = \frac{f}{2}(P_f V_f - P_i V_i)$$
$$= \frac{3}{2}(2P_0 \times 2V_0 - P_0 \times V_0) = \frac{9}{2}P_0 V_0$$

Work done in process $A \to B$ is equal to the Area covered by the graph with volume axis i.e.,

$$W_{A \to B} = \frac{1}{2}(P_0 + 2P_0) \times (2V_0 - V_0) = \frac{3}{2}P_0 V_0$$

Hence, $\Delta Q = \Delta U + \Delta W = \frac{9}{2}P_0 V_0 + \frac{3}{2}P_0 V_0 = 6P_0 V_0$

33. $\eta = 1 - \frac{T_L}{T_H}$

34. $\eta = \frac{T_1 - T_2}{T_1} - \frac{W}{Q} \Rightarrow Q = \left(\frac{T_1}{T_1 - T_2}\right)W$

$= \frac{600}{(600 - 300)} \times 800 = 1600$ J

35. $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = \frac{1}{5} \quad \because \eta = \frac{W}{Q}$

$\Rightarrow \quad \frac{1}{5} = \frac{W}{Q}$

$\Rightarrow \quad W = \frac{Q}{5} = \frac{6}{5} \times 10^4 = 1.2 \times 10^4$ J

36. Coefficient of performance $K = \frac{T_2}{T_1 - T_2}$

$= \frac{(273 - 23)}{(273 + 27) - (273 - 23)} = \frac{250}{300 - 250} = \frac{250}{20} = 5$

37. $\eta = \frac{T_1 - T_2}{T_1} = \frac{(273 + 727) - (273 + 227)}{273 + 727}$

$= \frac{1000 - 500}{1000} = \frac{1}{2}$

38. **(a)** $\frac{T'}{T} = \frac{3V}{V} = 3$

$T' = 3T = 3[40 + 273]K$

$= 3 \times 313$ K $= 939$ K

$T = (939 - 273)°$ C $= 666°$C

39.

(c) $\rho = \frac{PM}{RT}$

Density ρ remains constant when P/T or volume remains constant.

In graph (i) Pressure is increasing at constant temperature hence volume is decreasing so density is increasing. Graphs (ii) and (iii) volume is increasing hence, density is decreasing. Note that volume would had been constant in case the straight line in graph (iii) had passed through origin.

40. **(a)** Work done by gas in all four process is positive and in order

$$W_A > W_B > W_C > W_D$$

$\Rightarrow$ (c) is false

The change in initial energy U is same for all process.

$\therefore$ $Q_A = U + W_A$ (1)

$Q_B = U + W_B$ (2)

$Q_C = U + W_C$ (3)

$Q_D = U + W_D$ (4)

Hence $Q_A > Q_B > Q_C > Q_D$

$\Rightarrow$ (d) is false

from equations (1) and (4)

$$Q_A - Q_D = W_A - W_D$$

$\Rightarrow$ (a) is true

from equations (2) and (3)

$$Q_B - Q_C = W_B - W_C$$

$\Rightarrow$ (b) is false.

41.

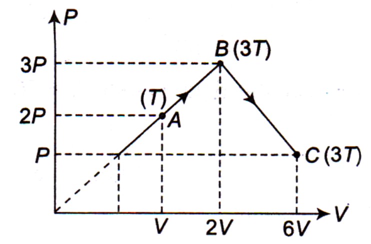
(a) $T = T_0 + aV^3 \Rightarrow \frac{PV}{nR} = T_0 + aV^3$

$$P = nR\left[\frac{T_0}{V} + aV^2\right]$$

For minimum P, $\frac{dP}{dV} = 0$

$$\Rightarrow \quad \frac{-T_0}{V^2} + a2V = 0 \Rightarrow V = \left(\frac{T_0}{2a}\right)^{\frac{1}{3}}$$

42. **(c)** If T is temperature of gas



at A, then $T_B = 3T = T_C$

In process AB, $\Delta W = \dfrac{5PV}{2}$

43. **(c)** Coefficient of performance

$$K = \frac{T_2}{T_1 - T_2} \Rightarrow 5 = \frac{(273-13)}{T_1 - (273-13)} = \frac{260}{T_1 - 260}$$

$\Rightarrow \quad 5T_1 - 1300 = 260 \quad \Rightarrow \quad 5T_1 = 1560$

$\Rightarrow \quad T_1 = 312\text{ K} \rightarrow 39°\text{C}$

44. **(c)** $\Delta Q = \Delta U + \Delta W$

$$\Rightarrow \quad \Delta W = (\Delta Q)_P - \Delta U = (\Delta Q)_P \left[1 - \frac{(\Delta Q)_V}{(\Delta Q)_P}\right]$$

$$= (\Delta Q)_P \left[1 - \frac{C_V}{C_P}\right] = Q = \left[1 - \frac{3}{5}\right] = \frac{2}{5}Q$$

$\because \ (\Delta Q)_P = Q$ and $\gamma = \dfrac{5}{3}$ for monatomic gas

45. (d) R is the universal gas constant